Nonlinear Gas Turbine Computer Modelling using NARMAX structures

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Abstract

This paper provides a comparison of techniques used to model the fuel flow to shaft speed relationship of a Spey gas turbine engine. Linear models are examined and the need for nonlinear modelling is justified. A technique based on nonparametric data analysis is proposed, to simplify the identification of a nonlinear model of the engine. A NARMAX model is identified and its performance validated against a range of small and large signal engine tests.

1. Introduction

Gas turbines were originally designed for aircraft propulsion but are now extensively used in aero, marine and industrial applications. With such widespread and increasing applications, the modelling of such engines is an issue of some importance.

Modelling of gas turbines is required both in the development and operational stages of an engine’s life. Design of control systems can be facilitated and, once the model has been verified against real engine data, a physical interpretation of the model parameters can be made. This allows initial assumptions about the engine characteristics to be checked.

This paper deals with the relationship between the fuel flow and shaft speed dynamics of an aircraft gas turbine. The shaft speeds are the primary outputs of a gas turbine, from which the internal pressures and the thrust can be calculated. Modern gas turbines usually have two shafts, one connecting a high pressure (HP) compressor to a HP turbine, the other connecting the low pressure (LP) compressor to a LP turbine. The Rolls Royce Spey Mk202 turbofan engine is an example of such an engine.

Although no longer in service, the Spey possess the same characteristics, for control purposes, as a modern engine such as that of the EJ200 Eurofighter [1].

Recent work by Evans [2,3,4] concentrated on testing the engine using small-amplitude multisine signals and then using frequency-domain techniques to identify linear models of high accuracy. The errors due to noise and nonlinearities were assessed and found to be small for these small-signal models. The same techniques were used to estimate models at a range of different operating points. Data were gathered under sea level static conditions at the Defence Evaluation & Research Agency (DERA) at Pyestock. Multisine and inverse repeat maximum length binary sequences (IRMLBS) were used at amplitudes of up to ±10% of the steady state fuel flow (±10% $W_f$).

However, all physical systems are invariably nonlinear, to a greater or lesser extent, so the need is apparent for a more complete gas turbine description using nonlinear models. Rodriguez [5,6] used a multiobjective genetic programming approach (MOGP) on the same data and allocated weights to various objectives to assess their significance in the identification of Nonlinear AutoRegressive Moving Average with eXogenous inputs (NARMAX) models of the engine. Higher-amplitude signals, such as triangular waves and three-level periodic signals with input amplitudes of up to ±40% $W_f$, were used to validate the estimated models. These caused the HP shaft speed to vary between 65% and 85% of its maximum value (%$N_{H}$).

In this paper an initial comparison is made between the linear models obtained using the MOGP approach and the models obtained using frequency-domain techniques. The need for a global nonlinear model is then motivated through a nonparametric analysis of the engine data. Knowledge gained from this analysis is used a priori to restrict the search space of nonlinear models under
consideration. Finally, a NARX model (NARMAX model with the noise terms excluded) of the HP shaft dynamics is estimated and its performance is illustrated on a number of different test signals.

2. Linear Gas Turbine modelling

Two methods are considered here for linear gas turbine modelling. The frequency domain identification (FDI) method provides a continuous s-domain model with pure time delay \( T_d \)

\[
H(s) = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_m s^m} e^{-j\omega T_d}
\]  

whereas the MOGP approach provides discrete models of the form

\[
y(k) = \sum_{i=1}^{n_y} a_i y(k-i) + \sum_{i=1}^{n_u} b_i u(k-i) + c
\]  

where \( y(k) \) and \( u(k) \) are the system output and input respectively; \( a_i, b_i \) and \( c \) being the coefficients of the model terms. Linear models were obtained across the engine operating range with both methods. These models can be validated by comparison of their frequency responses with the nonparametric frequency response function (FRF) of the engine. The basic input-output relationship in the frequency domain is given by

\[
H(j\omega) = \frac{Y(j\omega)}{U(j\omega)}
\]  

\( H(j\omega) \) being the FRF and \( Y(j\omega) \) and \( U(j\omega) \) the Fourier transforms of \( y(t) \) and \( u(t) \). The use of periodic signals allows the direct estimation of the FRF as the ratio of the mean values of the output and input coefficients, at the discrete test frequencies \( \omega_k \)

\[
\hat{H}(j\omega_k) = \frac{1}{M} \sum_{m=1}^{M} Y_m(j\omega_k) = \frac{\overline{Y}(j\omega_k)}{\overline{U}(j\omega_k)}
\]  

where \( M \) is the number of periods measured. This is termed the EV estimator and it has been shown that it is a maximum likelihood estimator if the input and output noises have a complex normal distribution, even if they are mutually correlated [7]. The frequency responses of the FDI and MOGP models were evaluated at a range of operating points and compared with the FRFs calculated using equation (4). Figure 1 provides an example of this comparison for the HP shaft models estimated at an operating point of 75\% \( N_{H} \), using an IRMLBS input signal of amplitude ±10\% \( W_f \).

![Figure 1. Frequency responses of HP shaft models using FDI (solid) and MOGP (dashed), with estimated FRF (crosses).](image)

It is clear that the MOGP model does not model the engine dynamics as well as the FDI model. This pattern was repeated across the range of operating points. Indeed, Rodriguez [5,6] points out that the linear MOGP models fail a number of validation tests and concludes that a nonlinear structure would be more appropriate.

It can be concluded that for linear models representing the small-signal dynamics of the gas turbine the frequency-domain identification of s-domain models is the best approach. This provides an accurate representation of the system, incorporating the pure time delay in the continuous model.

3. Detecting the nonlinearity

Tests were conducted at several operating points along the turbine running range from 53\% \( N_{H} \) to 89\% \( N_{H} \), at an input amplitude of ±10\% \( W_f \). The locations of the poles and zeros estimated using the FDI approach are shown in Figure 2 and some clear features can be deduced from the plot.

![Figure 2. Locations of poles and zeros estimated using the FDI approach.](image)

It is clear that the HP and LP shafts have different order dynamics. Cancelling pole-zero pairs suggest that the HP shaft is predominantly first-order, across most of the operating range, and that the LP shaft is second-order. It is also clear from Figure 2 that the position of the poles and zeros, in other words the dynamics of both shafts, change with the operating point. The dc gains of these models also decrease as the operating point is increased, as shown
in Table 1. This shows clearly that the gas turbine is nonlinear [8].

![Figure 2](image1.png)

Figure 2. Poles and zeros of the FDI models (a) HP shaft and (b) LP shaft. Showing poles (x) and zeros (o).

**Table 1.** Model dc gains at different operating points.

<table>
<thead>
<tr>
<th>Operating point</th>
<th>HP shaft dc gain</th>
<th>LP shaft dc gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>0.262</td>
<td>0.195</td>
</tr>
<tr>
<td>65</td>
<td>0.177</td>
<td>0.141</td>
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<tr>
<td>75</td>
<td>0.127</td>
<td>0.098</td>
</tr>
<tr>
<td>85</td>
<td>0.088</td>
<td>0.067</td>
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</table>

It is possible to detect the presence of this nonlinearity by analysing the data at a single operating point. If a signal contains only harmonics that are odd multiples of the fundamental (such as an IRMLBS or an odd multisine) then all the frequency contributions at the output resulting from any even-order nonlinearities will fall at even harmonics [9]. Thus the even nonlinearities can be detected just by inspection of the frequency content of the system input and output signals. Similarly if an odd-odd multisine is used (a signal where every other odd harmonic is also excluded) both even-order and odd-order nonlinearities can be detected, since the odd-order nonlinear contributions will fall at the omitted odd harmonics. A useful tool with which to assess the periodicity of the generated harmonics, and distinguish them from noise harmonics, is the squared coherence function

$$Y_n(\omega) = \frac{\frac{1}{M} \sum_{m=1}^{M} Y_m(j\omega)^2}{\frac{1}{M} \sum_{m=1}^{M} Y_m(j\omega) Y_m^*(j\omega)} = \frac{\overline{|Y(j\omega)|^2}}{G_{YY}(\omega)}$$  \hspace{1cm} (5)$$

where \(Y_m(j\omega)\) is the output spectrum at the excited and nonexcited frequencies, \(Y_m(j\omega)\) its complex conjugate and \(G_{YY}(\omega)\) the autospectrum of the output. The coherence represents the ratio of the periodic power to the total power at the output and if there is no periodic power at the output then the coherence will assume a value of \(1/M\). The periodic power due to nonlinearities can be detected since the coherence function at those frequencies will rise well above the \(1/M\) bound.

Evans [9] used the coherence function to detect the existence of an even-order nonlinearity in the engine. At the same time there was no evidence for the existence of odd-order nonlinearities. The nonlinear coherence of an IRMLBS at an input amplitude of ±10% \(W_f\) is plotted in Figure 3, along with the \(1/M\) bound, in order to assess the even-order nonlinear contributions. It is seen that the coherence of the even harmonics in the input spectrum is close to the \(1/M\) bound whereas the coherence of the even harmonics at the output is significant. This suggests the existence of a weak even-order nonlinearity in the engine. This even-order nonlinearity did not influence the estimated linear models due to the use of odd harmonic test signals.

![Figure 3](image2.png)

Figure 3. Nonlinear coherence of IRMLBS at (a) input and (b) output. Odd harmonics (solid), even harmonics (dashdot), 1/M bound (dashed).

The results of the small-signal tests thus point to the existence of an even-order nonlinearity in the engine. By examining the large signal tests this nonlinear effect should become even more apparent. A simple plot of the input-output relationship of a triangular wave is shown in Figure 4 and reveals the existence of nonlinearities in the system. A curve was fitted to the data and it can be seen from equation (6) that only the second-order nonlinear term is significant.
4. Nonlinear Gas Turbine modelling

Having detected the nonlinearity the need to develop a nonlinear model for the gas turbine is apparent. Leontaritis and Billings [10] introduced the NARMAX approach as a means of describing the input-output relationship of a nonlinear system. The model represents the extension of the well-known ARMAX model to the nonlinear case, and is defined as

\[ y(k) = F(y(k-1),...,y(k-n_y),u(k-1),... \]
\[ \ldots,u(k-n_u),e(k-1),...,e(k-n_e)) + e(k) \]  

(7)

where \( F \) is a nonlinear function; \( y(k), u(k) \) and \( e(k) \) represent the output, input and noise signals respectively; and \( n_y, n_u, \) and \( n_e \) are their associate maximum lags.

Billings and Tsang [11] used an orthogonal estimator for the identification of a NARMAX model. The orthogonal estimator is a very simple and efficient algorithm that allows each coefficient in the model to be estimated. At the same time, using a forward regression algorithm, it provides an indication of the contribution that the term makes to the system output. The disadvantage of this algorithm is that misleading information can arise regarding the significance of each term, since the cost function depends on the order in which the candidate terms are orthogonalised in the regression function. Rodriguez [6] developed a MOGP approach, which defines certain criteria and allocates weights to different objectives, to detect the structure of a NARMAX model.

This method was used to identify linear models for the engine and a family of NARX models, with the aim of describing both the small and large signal dynamics of the gas turbine.

In this paper \textit{a priori} knowledge of the engine nonlinear dynamics was used (obtained from section 3) to identify NARX models with the nonlinear terms restricted to the second order. Each model was tested with real data in order to arrive at the best nonlinear model capable of modelling both large and small signal engine dynamics. The quality of each model was assessed by a cost function that is simply the sum of the squares of the prediction errors, given by

\[ V = \sum_{k=1}^{N} (y(k) - \hat{y}(k|\theta_p))^2 \]  

(8)

where \( \theta_p \) is a vector of estimated coefficients, \( \hat{y}(t) \) indicates the one-step-ahead prediction output and \( y(t) \) is the measured output.

The signal used for estimation of the model parameters was an IRMLBS, a period of which is shown in Figure 5, with an input amplitude of \( \pm 10\% W_f \). Figure 6 shows a typical example of the spectrum of the measured fuel feed and the HP shaft speed. It can be seen that the signal-to-noise ratios are very good for this input amplitude, up to around 0.6 Hz.
Figure 6. Spectra of the IRMLBS test (a) input signal (b) system response.

The orthogonal estimator was applied to these data and several HP shaft models were obtained. Table 2 shows a selection of the model structures examined.

Table 2. Quadratic polynomial model structures

<table>
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<tr>
<th>Model terms</th>
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<tr>
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<td>202</td>
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</table>

It can be seen that the most significant terms are those in model 3, where the major drop in cost function occurs. Additional terms have little or no effect on the cost function. It must be noted here that the assessment of each model was only concluded after visual inspection of the model output as compared with the measured engine output. Nonparametric validation was then employed which consisted of the simulation of the individual models with measured inputs and comparison of the result with measured output. A time-domain comparison was employed for the high amplitude tests, whereas a frequency-domain comparison was employed for small-signal tests, since the periodicity of the small-amplitude IRLMBS signals allowed the direct estimation of the FRF according to equation (4). The best quadratic model obtained is shown in Table 3. Figure 7 shows a time-domain comparison between the model output and the measured gas turbine output for a small signal IRMLBS test, where it can be seen that a very good match is achieved.

Table 3. Quadratic polynomial model parameters

<table>
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<th></th>
<th>$c$</th>
<th>$y(k-1)$</th>
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<td>0.4006</td>
<td>0.5550</td>
<td>0.0049</td>
<td>0.0002</td>
<td>-7.10e-4</td>
</tr>
</tbody>
</table>

Figure 7 IRMLBS HP shaft test. (a) complete signal (b) portion of the signal. Model output (dotted) measured output (solid).

Figure 8. Frequency response of the HP shaft model (solid), with estimated FRF (crosses).

Figure 8 shows how the model frequency response matches the measured FRF, suggesting that this nonlinear
model is well capable of modelling the small-signal dynamics of the engine, as shown in both the frequency and time domains.

**Figure 9.** Triangular wave HP shaft test. Model output (dotted) measured output (solid).

The model was also validated against high-amplitude data. Figure 9 shows a triangular wave validation test where the model output follows the measured output very well. In this test the engine input was a triangular wave of period 100s and amplitude of $\pm 35\% \ W_f$. The next test shown in Figure 10 consists of a three-level sequence of period 100s with an input amplitude of $\pm 22\% \ W_f$. Again it can be seen that the model shows a good response.

**Figure 10.** Three-level HP shaft test. Model output (dotted) measured output (solid).

Finally, Figure 11 shows the output response to a triangular wave with a superimposed IRMLBS signal. The amplitude varied between $\pm 42\% \ W_f$ so as to cause the HP shaft speed to vary between 65% and 85% $N_{Hf}$. Again, the model output matches the real data very well, suggesting once more that the model derived in Table 3 is capable of modelling the high-amplitude dynamics of the gas turbine. These results can be extended to the LP shaft case, where a model of similar structure (but different parameter values) was estimated.

Having derived a nonlinear model for the engine, capable of modelling both high and low amplitude system dynamics, certain issues concerning the estimation of the model parameters should be considered. When estimating the NARX models, using different test signals gave widely differing results - most of which failed the cross-validation tests described above. For example, it was clear that in order to estimate a model capable of representing the small-signal engine dynamics a small-signal test should be used for estimation. The model presented in this paper was estimated using a small-signal IRMLBS test at 75% $N_{Hf}$.

McCormack [12] simulated different nonlinear models with different signals and concluded that broadband signals with a rich harmonic content provide superior parameter estimates for a variety of nonlinear models. This conclusion is confirmed by the results in this paper, where the best results were obtained using either small-signal IRMLBS tests or the triangular wave with superimposed IRMLBS signal.
5. Conclusions

A simple method to identify a nonlinear model of a gas turbine was proposed in this paper. This consisted of a nonparametric analysis of the engine data, in the time and frequency domains, to establish the existence and approximate order of the nonlinearity and the selection of a range of suitable NARX structures for the engine.

This allowed the search space of the potential NARX models to be considerably narrowed and facilitated the straightforward selection of an appropriate model structure. A model was estimated which performs well with both small-amplitude and high-amplitude tests.

The performance of the model was illustrated on a range of signals and shown to follow the output behaviour of the engine extremely well. Such a model could provide the basis for a global nonlinear controller of the engine. Some outstanding issues remain to be resolved with regard to the most appropriate signal design for nonlinear modelling, which are currently the subject of ongoing investigation. This paper illustrates how periodic test signals, frequency domain analysis and identification techniques, and time-domain NARMAX modelling can be effectively combined to enhance the modelling of an aircraft gas turbine.

6. Acknowledgements

This work was conducted on data gathered at the Defence Evaluation & Research Agency at Pyestock with the support of Rolls Royce plc. The authors would like to thank all the staff involved. Special thanks also to Katya Rodriguez-Vazquez for sharing her results, thus enabling a direct comparison between the two identification methods.

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